Abstract model theory for extensions of modal logic

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Many languages used in computer science (e.g., in knowledge representation, XML querying, system verification) are extensions of modal logic. But what does it mean to be an *extension of modal logic*? There are at least three different dimensions along which the basic modal logic can be extended:

1. Axiomatic extensions (or, restricting the class of frames) Traditionally, by an extension of the basic modal logic K, people have been referring to axiomatic extensions. Model theoretically, we can also see this as the study of modal logics of more specific frame classes. Many beautiful results have been proved for logics of specific frame classes. For instance, Spaan (1993) has shown than every non-trivial extension of S4.3 is NP-complete for satisfiability, and Marx and Venema (1997) have shown that every extension of the basic modal logic K with Sahlqvist axioms that have first-order universal Horn correspondents has Craig interpolation.

2. Language extensions (or, increasing the expressive power) A different way of extending modal logic is by making the language more expressive, e.g., by adding a universal modality, backward looking modalities, the $\downarrow$-binder of hybrid logic, etc. A fundamental theorem here is of course the Van Benthem bisimulation theorem, which tell us that the basic modal language is the bisimulation variant fragment of first-order logic. Some recent results give a nice insight into what properties extensions of the basic modal language may have:

- Van Benthem (2007) proved that if an extension of the basic modal language satisfies bisimulation invariance and compactness, then it is not really more expressive than the basic modal language. In other words, every proper extension of the basic modal language must lack either compactness or bisimulation invariance. More results along this line can be found in (Ten Cate, Van Benthem and Väänänen, 2007).
- Ten Cate (2005) considered $M(D)$, the extension of the basic modal language with the difference modality, which is well known to lack Craig interpolation, and showed that the least expressive extension of $M(D)$ with Craig interpolation is full first-order logic. Likewise, it was shown there that the hybrid language $H(@, \downarrow)$ is the least expressive extension of $H(@)$ with Craig interpolation.

3. Signature extensions (or, changing the type of structures considered) Besides Kripke structures, modal logic can be used to describe other types of structures, e.g., topological spaces, neighborhood models, or even theories of arithmetic. Moreover, there is no need to stick to the rigid format of unary modalities: some of these types of structures naturally give rise to k-ary modalities with $k > 1$. Recently a general framework has arisen, in which many of these "signature extensions" can be accounted for, based on coalgebra. Kripke models (as well as topological spaces, probabilistic transition systems, and polyadic Kripke models) are examples of coalgebras for well behaved functors on the category of sets. It was recently shown that certain results on modal logic can be generalized to whole families of such functors, yielding for instance a general uniform interpolation result (Kupke and Venema, 2005) and a general Goldblatt-Thomason result (Kurz and Rosicky, 2007).

In this talk I will discuss these three dimensions, with a special focus on the second, and I will discuss some interesting interactions between them.